Generation and Proof of Elliptically Polarized X-Rays

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Dedicated to Herrn Prof. G. Hildebrandt on the occasion of his 60th birthday

Using an arrangement being analogous to optics of visible light the phase relation of the mutually perpendicularly polarized wave fields are examined in the Laue case of X-ray diffraction. Bragg reflections at an angle of reflection of about 45° are used as polarizer and analyzer. The different phase relations result from different thicknesses of a wedge-shaped silicon crystal, which is placed between the polarizer and the analyzer and adjusted for the symmetric 220-$\mathrm{Cu}_{K\alpha 1}$-Laue case. The determined polarization states produced by coherent excitation of both σ-polarized and π-polarized waves in the silicon crystal coincide very well with calculations of the dynamical theory of X-ray diffraction.

1. Introduction

In the case of simple transmission, polarization phenomena are generally not observed with X-rays. Such phenomena, however, may occur with crystal diffraction because of the different behaviour of differently polarized X-rays. From the dynamical theory of X-ray diffraction, cf. v. Laue [1], it can be concluded that in the two beam case four wave fields are generated in the crystal by excitation with unpolarized X-rays.

For two of them the electric vector lies in the plane of reflection (π-polarization) and for the other two the electric vector is normal to this plane (σ-polarization). Since the wave vectors of these wave fields are different, interference phenomena known as the Pendellösung fringes of the two σ-polarized and the two π-polarized wave fields are observed in the Laue case. The fading of fringes arises from the difference of the Pendellösung periods of the σ-polarized and π-polarized X-rays, cf. Hart and Lang [2] and Hattori, Kuriyama, and Kato [3]. In case of coherent excitation of the σ-polarized and the π-polarized waves by one linearly polarized X-ray beam, whose electric vector is inclined by 45° with respect to the reflecting plane, the existence of elliptically polarized X-rays can be concluded from the dynamical theory of X-ray diffraction, cf. Skalicky and Malgrange [4]. The direct proof of the phase shift between the coherently excited σ-polarized and π-polarized waves is the subject of this paper. A polarization-optical analyzer is used in order to study the phase relation between the mutually perpendicularly X-rays. First of all, however, the phase differences and the corresponding polarization states in the performed experiment are calculated with regard to the absorption.

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2. Theoretical Considerations

In the symmetric Laue case the phase difference between the anomalously weakly absorbed σ-polarized and π-polarized waves is given by

$$\Delta_1 = 2\,\pi\,(K_{\mathrm{H1}}^{\sigma} - K_{\mathrm{H1}}^{\pi})\,t\,, \tag{1}$$

where t is the crystal thickness and $K_{\mathrm{H1}}^{\sigma,\pi}$ represents the respective amount of the wave vector of the diffracted σ-polarized and π-polarized waves. If absorption is neglected the amplitude ratio of the mutually perpendicularly polarized waves is proportional to

$$D_{\mathrm{H1}}^{\sigma}/D_{\mathrm{H1}}^{\pi} \sim \exp(i\,\Delta_1)\,, \tag{2}$$

where $D_{\mathrm{H1}}^{\sigma,\pi}$ is the amount of the electric displacement. Thus the periodicity Ω of the phase difference between the σ-polarized and the π-polarized waves is given by

$$\Omega = 1/(K_{\mathrm{H1}}^{\sigma} - K_{\mathrm{H1}}^{\pi})\,. \tag{3}$$

The phase difference between the anomalously strongly absorbed wave fields corresponds with the phase difference between the anomalously weakly absorbed wave fields, but with opposite sign. This means that two elliptically polarized waves (in borderline cases linearly and circularly polarized ones, respectively) with the same amplitude and excentricity, but with opposite sense of rotation always exist in the crystal independently of its thickness. The interference of these waves gives a linearly polarized wave. Because of the anomalous absorp-

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tion, however, the share of the strongly absorbed wave fields in the reflected intensity decreases with increasing crystal thickness. Hence the oscillations of the Pendellösung fringes vanish. Thus in the experiment elliptically polarized X-rays can only be generated due to the phenomenon of the anomalous absorption.

The periodicity of the phase difference between the σ-polarized and the π-polarized waves is given by $\Omega = 97.5\ \mu m = t_1$ in the symmetric 220-$Cu_{K\alpha 1}$-Laue case in silicon. Circularly polarized X-rays are generated at a crystal thickness of $(5/4)\,\Omega = 121.9\ \mu m = t_2$, if merely the wave field 1 is considered. $\Delta_1 = 2\pi$ and $\Delta_2 = (5/2)\pi$ are the according values of the phase differences. But at these thicknesses the anomalously strongly absorbed wave fields also contribute to the reflected intensity and in addition to that the absorption coefficients of the σ-polarized and the π-polarized waves differ slightly. On the assumption that the σ-polarized and the π-polarized waves are equally strongly excited by a plane wave the amplitudes of the electric displacement were calculated according the following equation, cf. [1]

$$|D_H^{\sigma,\pi}|^2 = \frac{\gamma_0\,|\chi_H|\exp[-\frac{1}{2}(\sigma_1+\sigma_2)\,t]}{\gamma_H\,|\chi_{\bar{H}}|\,4\cosh^2 v_r} \cdot \{\exp[\tfrac{1}{2}(\sigma_1-\sigma_2)\,t] + \exp[-\tfrac{1}{2}(\sigma_1-\sigma_2)\,t] - 2\cos[K(\delta_{1r}-\delta_{2r})\,t]\}\,|D_0^{inc}|^2, \tag{4}$$

where

D_0^{inc}	amplitude of the displacement of the incident wave,
γ_0, γ_H	direction cosine of the incident and diffracted beam, respectively,
σ_1, σ_2	absorption coefficient of the anomalously weakly and anomalously strongly absorbed wave field, respectively,
δ_{1r}, δ_{2r}	real part of accomodation of the wave fields,
v_r	angular coordinate,
K	wave vector of the incident wave.

Since the σ-polarized and the π-polarized waves are coherently excited the amplitude distribution $A(\varphi)$ which results from the crystal thicknesses t_1 and t_2, respectively, can be calculated,

$$A(\varphi) = \sqrt{(D_H/D_0)^2\cos^2(\varphi-\Delta) + (D_H/D_0)^2\cos^2\varphi}\,, \tag{5}$$

where the maximum of the function $A(\varphi)$ was normalized. The function $A(\varphi)$ is shown in Fig. 3 (full lines).

3. Experiments

The scheme of the experimental arrangement is shown in Figure 1. A 111-surface oriented germanium crystal plate was taken as the polarizer using the asymmetric 511-Bragg case ($\theta_B = 45.08°$). A wedge-shaped silicon single crystal adjusted in the

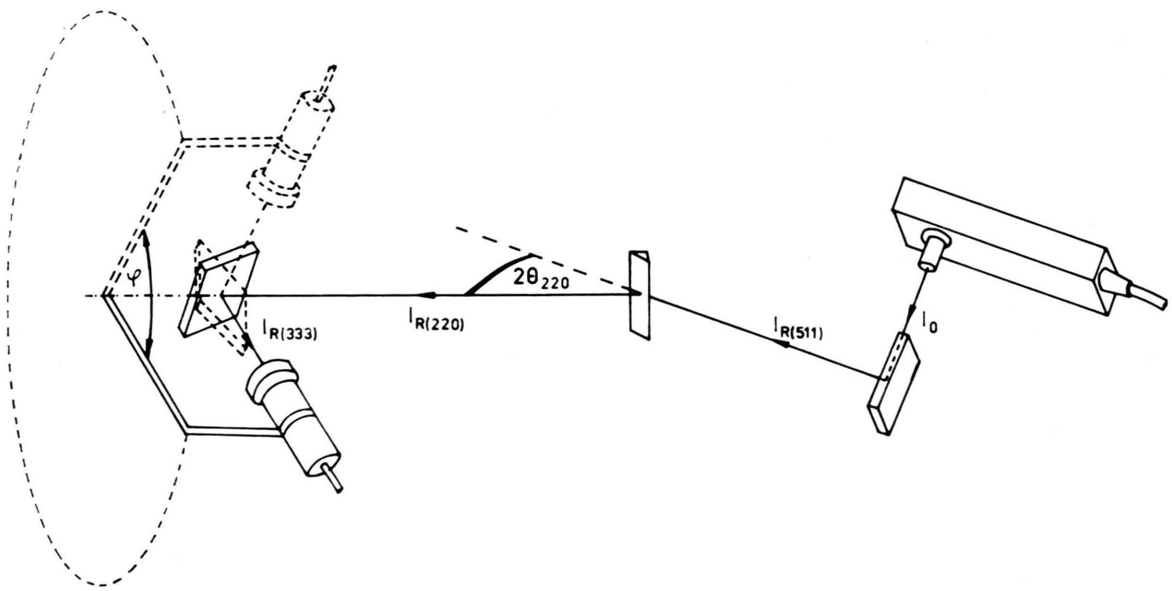

Fig. 1. Scheme of the experimental arrangement.

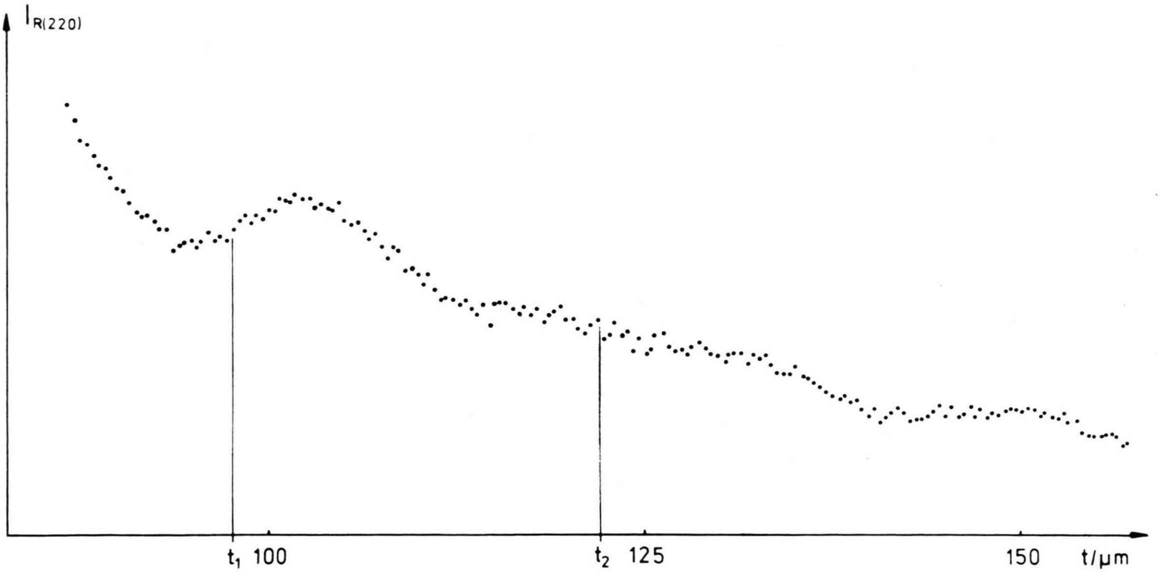

Fig. 2. The dependence of the intensity $I_{R(220)}$ (arbitrary units) on the crystal thickness t.

symmetric 220-Laue case serves as a phase shifter for the incident linearly polarized X-rays. The base surface of the wedge-shaped crystal is parallel to the 110-lattice planes; the magnitude of the wedge angle is about one degree. The adjustment was made in this way that the reflecting plane of the polarizer was inclined by 45° with respect to the reflecting plane of the phase shifter. Thus the σ-polarized and the π-polarized waves were coherently and equally strongly excited. Furthermore the excited thickness range had to be limited for a unique phase relation. The limitation was performed by a 300 μm wide slit. This means that a thickness range of about 5 μm was excited implying a phase difference range of about $\pi/10$. The wedgeshaped crystal was mounted on a Lang camera so that the diffracted intensity $I_{R(220)}$ could be measured in dependence on the crystal thickness t. The measured curve is shown in Figure 2.

A further reflection, the symmetric 333-Bragg case ($\theta_B = 45.08°$), was used for the analysis of the polarization states of the X-ray beam $I_{R(220)}$. This germanium crystal plate could be rotated about an axis parallel to the incident beam. The rotation was measured by the angle φ. This rendered it possible to determine the polarization state of the incident beam by measuring of its components. In the case of a noncoplanar ray path the integral intensity had to be measured because of the influence of dispersion and divergence. Using a highly sensitive angular setting, cf. Brümmer, Höche, and Eisenschmidt [5], it was possible to vary the angle of incidence for measurements of the entire reflection curves at a fixed angle φ. For the crystal thicknesses t_1 and t_2 the dependence of the integral intensity on the angle φ was determined. The results are shown in Fig. 3 (points). The angle φ was changed in steps of 10 degrees over an interval of 180 degrees.

4. Results

In the dependence on the thickness t of the wedge-shaped silicon crystal adjusted in the symmetric 220-Laue case the integral reflected intensity is shown in Figure 2. The oscillations and the fading region can be seen clearly. The calibration of the abscissa axis results from comparison of the experimental curve with the calculated one. For both crystal thicknesses t_1 and t_2, which correspond to a phase difference between the σ-polarized and the π-polarized waves of zero and $\pi/2$, respectively, the polarization states were determined as described above. The background-corrected integral intensities I_R are shown in Fig. 3 (points). For a comparison of these values with the amplitude distribution the calculated polarization states of the amplitudes must be convoluted with the $\cos^2\varphi'$-function of the analyzer according to the Law of Malus. The convoluted curves are also shown in Fig. 3 (broken lines). Due to the wedge-shaped design of the phase shifter and

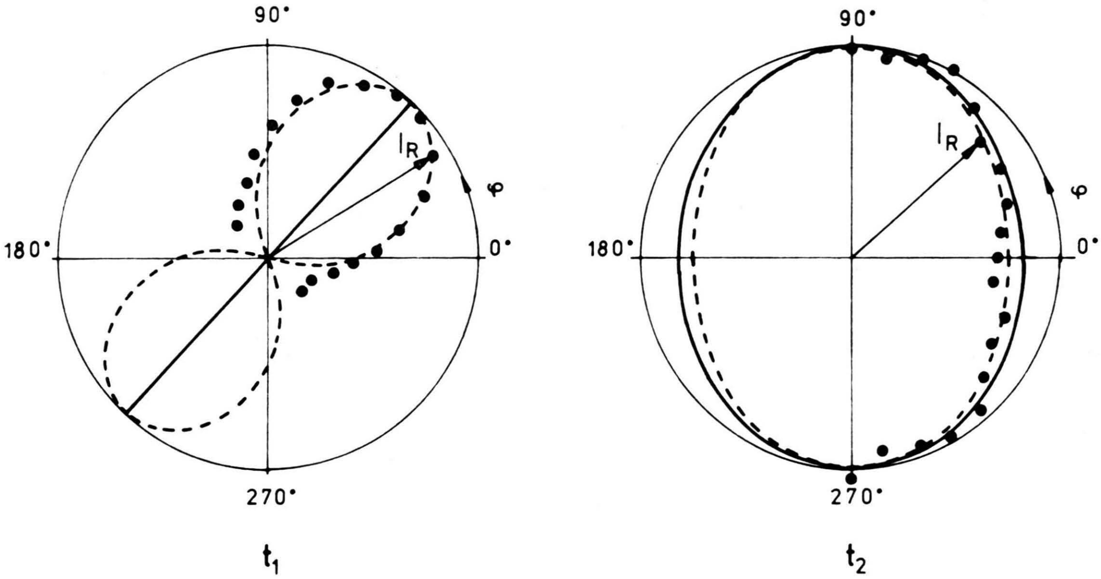

Fig. 3. The polarization states of the beam $I_{R(220)}$ corresponding to the crystal thicknesses t_1 and t_2. $\varphi = 0°$ and $\varphi = 180°$ correspond to the values of the π-polarized waves, $\varphi = 90°$ and $\varphi = 270°$ correspond to the σ-polarized ones. Full lines — Calculated amplitude distribution. Broken lines — With $\cos^2 \varphi'$ — function convoluted amplitude distribution. Points — Experimentally determined values.

the necessary width of the slit not only one definite crystal thickness, but a thickness range was excited. Taking this into account the experimentally determined values agree to the calculated ones very well.

5. Summary

Using a non-coplanar three-crystal arrangement the phase relation between the σ-polarized and the π-polarized wave fields was examined in the Laue case. In particular the thickness-dependent phase difference between the σ-polarized and the π-polarized waves of the diffracted X-rays and therefore the existence of elliptically polarized X-rays was directly proved. The phenomenon of the anomalous absorption rendered it possible. The experimental results confirm the predictions of the dynamical theory of X-ray diffraction accurately.

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